

Exercise 33Find $f'(a)$.

$$f(t) = \frac{2t + 1}{t + 3}$$

SolutionDetermine the derivative of $f(t)$.

$$\begin{aligned} f'(t) &= \lim_{h \rightarrow 0} \frac{f(t+h) - f(t)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{2(t+h)+1}{(t+h)+3} - \frac{2t+1}{t+3}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{2t+2h+1}{t+h+3} - \frac{2t+1}{t+3}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{(2t+2h+1)(t+3)}{(t+h+3)(t+3)} - \frac{(2t+1)(t+h+3)}{(t+3)(t+h+3)}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{(2t+2h+1)(t+3) - (2t+1)(t+h+3)}{(t+h+3)(t+3)}}{h} \\ &= \lim_{h \rightarrow 0} \frac{(2t + 2h + 1)(t + 3) - (2t + 1)(t + h + 3)}{h(t + h + 3)(t + 3)} \\ &= \lim_{h \rightarrow 0} \frac{(2t^2 + 6t + 2ht + 6h + t + 3) - (2t^2 + 2th + 6t + t + h + 3)}{h(t + h + 3)(t + 3)} \\ &= \lim_{h \rightarrow 0} \frac{5h}{h(t + h + 3)(t + 3)} \\ &= \lim_{h \rightarrow 0} \frac{5}{(t + h + 3)(t + 3)} \\ &= \frac{5}{(t + 3)(t + 3)} \\ &= \frac{5}{(t + 3)^2} \end{aligned}$$

Plug in $t = a$ to this formula to get $f'(a)$.

$$f'(a) = \frac{5}{(a + 3)^2}$$